

Dynamos driven by weak thermal convection and heterogeneous outer boundary heat flux



Swarandeeep Sahoo^{a,*}, Binod Sreenivasan^{a,*}, Hagay Amit^b

^a Centre for Earth Sciences, Indian Institute of Science, Bangalore 560012, India

^b CNRS UMR 6112, Université de Nantes, Laboratoire de Planétologie et de Géodynamique, 2 rue de la Houssinière, Nantes F-44000, France

ARTICLE INFO

Article history:

Received 20 April 2015

Received in revised form 28 August 2015

Accepted 17 November 2015

Available online 2 December 2015

Keywords:

Geodynamo

Early Earth

Geomagnetic field

Core–mantle interaction

Helicity generation

ABSTRACT

We use numerical dynamo models with heterogeneous core–mantle boundary (CMB) heat flux to show that lower mantle lateral thermal variability may help support a dynamo under weak thermal convection. In our reference models with homogeneous CMB heat flux, convection is either marginally supercritical or absent, always below the threshold for dynamo onset. We find that lateral CMB heat flux variations organize the flow in the core into patterns that favour the growth of an early magnetic field. Heat flux patterns symmetric about the equator produce non-reversing magnetic fields, whereas anti-symmetric patterns produce polarity reversals. Our results may explain the existence of the geodynamo prior to inner core nucleation under a tight energy budget. Furthermore, in order to sustain a strong geomagnetic field, the lower mantle thermal distribution was likely dominantly symmetric about the equator.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The geodynamo is powered by thermochemical convection of an electrically conducting fluid in the outer core. Thermal convection originates from secular cooling due to the loss of heat through the core–mantle boundary (CMB), latent heat release at the inner-core boundary (ICB) and possibly radiogenic heating within the shell volume. Chemical convection originates from the release of light elements at the ICB as the core freezes.

Evidence for the existence of a geomagnetic field goes back to the Hadean period (Tarduno et al., 2015). Estimates of the age of the inner core are less than 1 Gyr (Nakagawa and Tackley, 2013). It is therefore likely that the geodynamo has operated in its early stages, that is, prior to inner core nucleation, on purely thermal convection. However, large estimates of the thermal conductivity of the outer core (de Koker et al., 2012; Pozzo et al., 2012, 2013; Hirose et al., 2013) suggest that the thermal gradient in the core is near, or even below the adiabat. At present the geodynamo is predominantly powered by the release of light elements due to inner core freezing (Olson, 2007); however, before inner core nucleation, it is not clear how the early geodynamo was sustained under such tight energetic constraints.

Convection in the core may be affected by buoyancy flux heterogeneities at its outer boundary. It has been shown that heteroge-

neous CMB heat flux may determine the long-term pattern of the geomagnetic field on the CMB (Bloxxham, 2002; Olson and Christensen, 2002; Gubbins et al., 2007; Willis et al., 2007; Amit et al., 2010), the flow at the top of the core (Aubert et al., 2007), and the ICB buoyancy flux (Aubert et al., 2008; Amit and Choblet, 2009; Gubbins et al., 2011). Core–mantle boundary heterogeneity may also affect the dynamo onset. While it has been proposed that thermal winds driven by the lower mantle heterogeneity can enhance dynamo action (Sreenivasan, 2009; Aurnou and Aubert, 2011; Dietrich and Wicht, 2013), the applicability of these models for early Earth is debatable because of the large lateral variations in heat flux required to obtain a significant magnetic energy (Aurnou and Aubert, 2011) or because convection in these studies is not purely thermal as in early Earth's core (Sreenivasan, 2009; Aurnou and Aubert, 2011). In addition, in these studies a large inner core consistent with present-day core geometry is used.

In this paper we analyze numerical dynamo models powered by purely thermal convection with moderate heat flux variations imposed on the outer boundary. The size of the inner core is kept very small. We examine the impact of different CMB heat flux patterns and amplitudes on the dynamo onset. Finally, possible application to early Earth conditions is discussed.

2. Method

We consider an electrically conducting fluid confined between two concentric, co-rotating spherical surfaces. For numerical sta-

* Corresponding author.

E-mail address: bsreeni@ceas.iisc.ernet.in (B. Sreenivasan).

bility, we retain a small conducting inner sphere of radius 0.1 times the outer sphere radius. The principal body forces acting on the fluid core are the thermal buoyancy force modulated by the lateral thermal variations at the outer boundary, the Coriolis force originating from the background rotation of the system and the Lorentz force arising from the interaction between the induced electric currents and the magnetic fields. The governing equations are in the Boussinesq approximation (Kono and Roberts, 2002). Lengths are scaled by the thickness of the spherical shell L , and time is scaled by the magnetic diffusion time, L^2/η , where η is the magnetic diffusivity. The temperature is scaled by βL^2 , where β is a constant proportional to the uniform volumetric heat source S (see below), the velocity field $\mathbf{u}$ is scaled by η/L and the magnetic field $\mathbf{B}$ is scaled by $(2\Omega\rho\mu\eta)^{1/2}$ where Ω is the rotation rate, ρ is the fluid density and μ is the free space magnetic permeability. The scaled magnetic field, known as the Elsasser number Λ , is an output derived from the volume-averaged magnetic energy in our dynamo simulations. The role of CMB heterogeneity in dynamo action is studied by imposing prescribed heat flux patterns on the outer boundary. Purely thermal convection is modelled by imposing zero heat flux on the inner boundary, so although the inner core size is non-zero, it is passive in terms of core convection. Previous systematic parametric studies of numerical dynamos find that a small and passive inner core has little effect on dynamo models (Aubert et al., 2009; Hori et al., 2010) even with the no-slip condition on the ICB. Although a small inner core might prevent equator-crossing meridional flow which can exist in highly supercritical convection (Landeau and Aubert, 2011), our models operate in a distinctive parameter regime close to convective onset, where rotational effects are dominant and the flow avoids crossing the equatorial plane.

The non-dimensional magnetohydrodynamic (MHD) equations for the velocity $\mathbf{u}$, magnetic field $\mathbf{B}$ and temperature T are

$$EPm^{-1} \left(\frac{\partial \mathbf{u}}{\partial t} + (\nabla \times \mathbf{u}) \times \mathbf{u} \right) + \hat{\mathbf{z}} \times \mathbf{u} = -\nabla p^* + RaPmPr^{-1} T \mathbf{r} + (\nabla \times \mathbf{B}) \times \mathbf{B} + E \nabla^2 \mathbf{u}, \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = PmPr^{-1} \nabla^2 T + S, \quad (3)$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{B} = 0, \quad (4)$$

The modified pressure p^* in Eq. (1) is given by $p + \frac{1}{2}EPm^{-1}|\mathbf{u}|^2$, where p is the fluid pressure. The velocity satisfies the no-slip condition at the boundaries and the magnetic field matches a potential field at the outer boundary. The dimensionless parameters in Eqs. (1)–(3) are the Ekman number $E = \nu/2\Omega L^2$ which measures the ratio

of viscous to rotational forces, the Prandtl number $Pr = \nu/\kappa$ which is the ratio of viscous to thermal diffusivities, the magnetic Prandtl number $Pm = \nu/\eta$ which is the ratio of viscous to magnetic diffusivities, and a modified Rayleigh number Ra which is the product of the classical Rayleigh number and the Ekman number given by $g\gamma\beta L^3/2\Omega\kappa$, where g is the gravitational acceleration acting radially inward, γ is the coefficient of thermal expansion, β is the scaling constant for temperature and κ is the thermal diffusivity. The last control parameter is the amplitude of the outer boundary heat flux heterogeneity defined by its peak-to-peak difference normalized by the mean:

$$q^* = \frac{q_{\max} - q_{\min}}{q_0} \times 100\%. \quad (5)$$

Note that defined this way, locally inward superadiabatic heat flux would occur for $q^* > 200\%$.

For the majority of our simulations, we choose $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra = 1.2Ra_c$, where Ra_c is the critical Rayleigh number for onset of non-magnetic convection (with homogeneous outer boundary heat flux). A few runs are performed at $E = 1.2 \times 10^{-5}$, $Pr = 1$, $Pm = 10$ and $Ra = 1.8Ra_c$. Finally, we explore a parameter regime with $Ra = 0.94Ra_c$ at $E = 1.2 \times 10^{-4}$ and $Pm = 50$ to study dynamo onset when there is no convection with homogeneous outer boundary heat flux. Approaching dynamo onset at low Rayleigh numbers and numerically accessible Ekman numbers necessitates a large electrical conductivity, which is why Pm is set to a high value. This problem is common to practically all dynamo models, which operate with $Pm \sim 1 - 10$ values (e.g. Christensen and Aubert, 2006), much larger than the core value of $Pm \sim 10^{-6}$ (Olson, 2007). It is, however, possible that $Pm \sim 1$ can be eventually reached in calculations at progressively lower E , which are computationally far more expensive. Despite the artificial enhancement of viscous diffusion due to our choice of internal parameters, our model may effectively capture the dynamics of the rapidly rotating core affected by lateral CMB heat flux variations.

The basic state buoyancy profile is obtained by solving the energy Eq. (3) under steady state and no flow conditions:

$$PmPr^{-1} \nabla^2 T + S = 0, \quad (6)$$

where the uniform volumetric heat source S , assumed to be $3PmPr^{-1}$ in this study, mimics secular cooling and radiogenic heat sources. Eq. (6) is then solved for the non-dimensional basic state heat flux $\partial T/\partial r$, using the zero flux condition at the inner boundary.

The dynamo calculations at $E = 1.2 \times 10^{-4}$ and $Pm = 50$ are performed with 108 Chebyshev collocation points in radius and a spherical harmonic degree cut-off value of $l = 108$. For $E = 1.2 \times 10^{-5}$ and $Pm = 10$, 144 radial grid points and a spectral

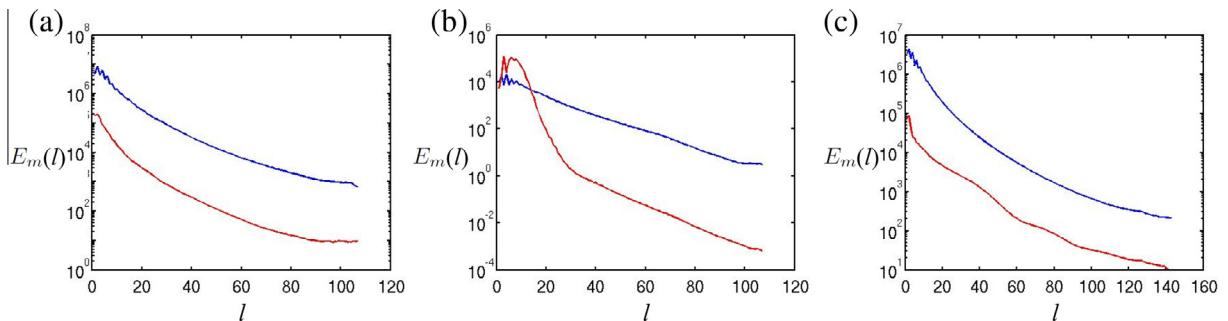


Fig. 1. Time-averaged kinetic (red) and magnetic (blue) energy versus spherical harmonic degree l . (a) and (b): Dynamos at $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 1.2$ with Y_1^1 ($q^* = 60\%$) and Y_2^2 ($q^* = 60\%$) boundary heat flux patterns respectively. (c) Dynamo at $E = 1.2 \times 10^{-5}$, $Pr = 1$, $Pm = 10$ and $Ra/Ra_c = 1.8$ with Y_1^1 ($q^* = 60\%$) heat flux pattern respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

cut-off of $l = 144$ are used. Fig. 1(a)–(c) show that the kinetic and magnetic energies decay by at least 4 orders of magnitude with l in our calculations with heterogeneous boundary heat flux in two different parameter regimes. This demonstrates that the spatial resolution of the dynamo models is adequate.

3. Results

The critical Rayleigh number Ra_c (with homogeneous outer boundary heat flux) is determined by searching for the minimum Rayleigh number for onset of non-magnetic convection. Fig. 2 shows that convection at $E = 1.2 \times 10^{-4}$ and $Pr = 1$ sets in at $Ra = 72$. In this regime we explore dynamo onset with heterogeneous boundary heat flux for both $Ra = 85$ ($Ra = 1.2Ra_c$) and $Ra = 68$ ($Ra = 0.94Ra_c$). When a lateral variation in heat flux is imposed on the outer boundary of the model, the magnetic field evolves in a way that depends on the symmetry of the boundary heterogeneity about the equator and its magnitude. Fig. 3 shows the magnetic field distribution at the outer radius for two patterns of boundary heat flux variation – an equatorially symmetric Y_1^1 pattern and an equatorially antisymmetric Y_2^1 pattern, both with $q^* = 60\%$. In either case, the magnetic flux is concentrated in regions where the boundary condition enhances the outward heat flux. The anti-symmetric Y_2^1 pattern results in a weaker axial dipole field (compare the scales of Fig. 3(b) and (d)).

3.1. The role of boundary heterogeneity in dynamo onset

In general, equatorially symmetric boundary heat flux variations with moderate q^* smaller than unity generate non-reversing magnetic fields (Figs. 4(a)–(d); Tables 1–3). For a relatively large q^* , the magnetic field grows and saturates into an equilibrated state. One calculation with a stress-free inner boundary (see Fig. 4(b)) confirms that changing the flow condition on the small inner core has practically no effect on dynamo action. For intermediate q^* , the magnetic energy varies at a very slow rate (see, for example, the 30% runs, shown in green, for the Y_1^1 and

Y_2^2 variations in Fig. 4(a) and (b) respectively). For much smaller q^* , however, the seed magnetic field simply decays to zero. The runs with $Ra < Ra_c$ display a very similar trend (Fig. 4(d)), implying that CMB lateral variations can support the dynamo even when there is no convection in the reference state with homogeneous CMB heat flux.

As q^* is increased beyond the minimum value for dynamo onset, the time variation of the magnetic field progressively decreases. For example, the standard deviations of the magnetic energy for the Y_1^1 variation are $\{1.05, 0.46, 0.26\} \times 10^7$ for q^* of 40%, 72% and 90% respectively, consistent with earlier studies that show suppression of time variations in magnetic energy as q^* is increased for $Ra \sim Ra_c$ in numerical dynamos (Willis et al., 2007; Sreenivasan, 2009).

The effect of the equatorially anti-symmetric variation Y_2^1 is different in character from the symmetric variations in that, when the dynamo sets in there are accompanying polarity reversals of the axial dipole (see the cases $q^* = 60\%$ and 65%). From the spiked trend of the magnetic energy for the 60% variation (Fig. 4(e), blue), we find that the field intensity in the shell volume fluctuates between 0.14 and 1.67, the high values being attained just before a reversal and the low values during a reversal. Fig. 4(f) gives the reversal pattern of the axial dipole (blue) over 2 magnetic diffusion times. During chrons the axial dipole dominates the field spectrum on the CMB, as in the reversing dynamos found by Olson et al. (2011) in an Earth-like transitional parameter regime without boundary heterogeneity. The dipole field intensity on the CMB (red) peaks just before a reversal and vanishes when the dipole axis crosses the equatorial plane. As no harmonic other than Y_1^1 flips polarity, the axial dipole intensity falls below that of other harmonics during a reversal. Fig. 5 shows the radial magnetic field distribution at the outer boundary during a reversal that spans approximately 0.07 magnetic diffusion times. As our reference homogeneous CMB heat flux model does not even produce a dynamo, we conclude that the reversals in the Y_2^1 model are triggered by the CMB heterogeneity itself, in contrast to earlier models (Glatzmaier et al., 1999; Kutzner and Christensen, 2004; Olson et al., 2010; Olson and Amit, 2014) where the role of the lower mantle heterogeneity was to modify the frequency of reversals that already existed in the reference homogeneous CMB dynamos.

To understand how the lower mantle heterogeneity supports the dynamo, we must examine the amplitudes of the flows that are generated in the spherical shell in response to the boundary variations. Tables 1–3 reveal that the value of the magnetic Reynolds number Rm , classically used to mark the threshold for dynamo onset (Moffatt, 1978), generally increases with q^* , but unfortunately does not help determine a threshold for dynamo action. Beyond a critical value of q^* there is a sharp increase in the strength of magnetic field, with the magnetic energy significantly higher than the kinetic energy. For the Y_2^2 pattern, this value is approximately 50%, whereas for Y_1^1 it is 40%. The precise numerical values of q^* are not so important as they depend on the dynamo model internal parameters. The fact that a 30% Y_1^1 variation causes the seed field to decay at a slow rate, whereas the slightly higher 40% case generates a strong magnetic field (a ratio of magnetic to kinetic energies of 40; see Table 1), points to an underlying boundary-induced dynamical criterion for dynamo onset.

3.2. A criterion for dynamo onset

As the seed field could not have produced a Lorentz force large enough to change the flow substantially, for example in early Earth, the dynamics in the early growth phase of the magnetic field

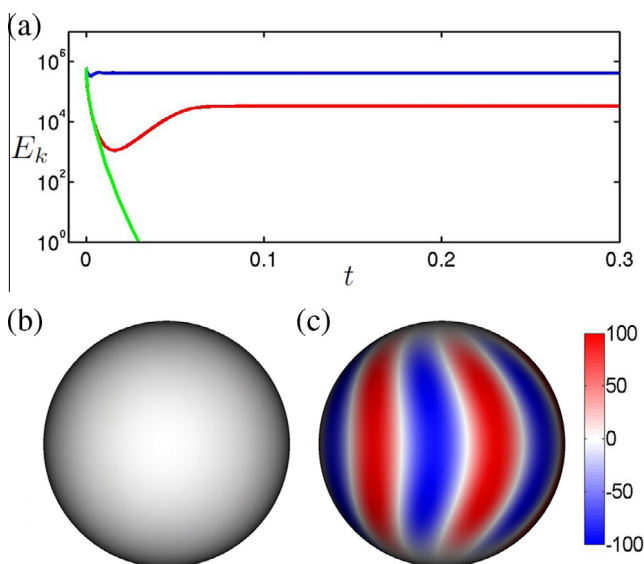


Fig. 2. (a) Total kinetic energy (E_k) versus time for non-magnetic runs at $E = 1.2 \times 10^{-4}$ and $Pr = 1$. The cases shown are $Ra = 68$ (green), 72 (red) and 85 (blue), $Ra = 72$ being the critical value for onset of convection. (b) and (c): Radial velocity on a spherical surface of radius $r = 0.8$ for $Ra = 68$ and $Ra = 72$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

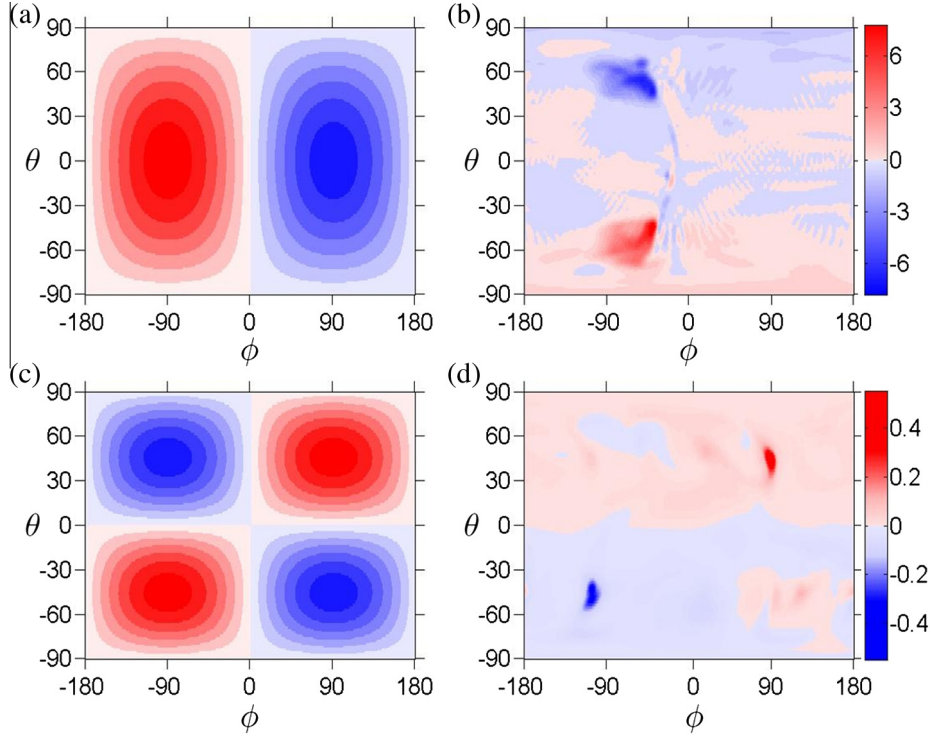


Fig. 3. (a) Equatorially symmetric Y_1 CMB heat flux pattern. (b) Snapshot of the radial magnetic field at the CMB in the dynamo with pattern (a) and $q^* = 60\%$. (c) Equatorially anti-symmetric Y_2 CMB heat flux pattern. (d) Radial magnetic field at the CMB in the dynamo with pattern (c) and $q^* = 60\%$. The dynamos are run with the parameters $E = 1.2 \times 10^4$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 1.2$. The latitude and longitude are denoted by θ and ϕ . Positive (negative) values are in red (blue) respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

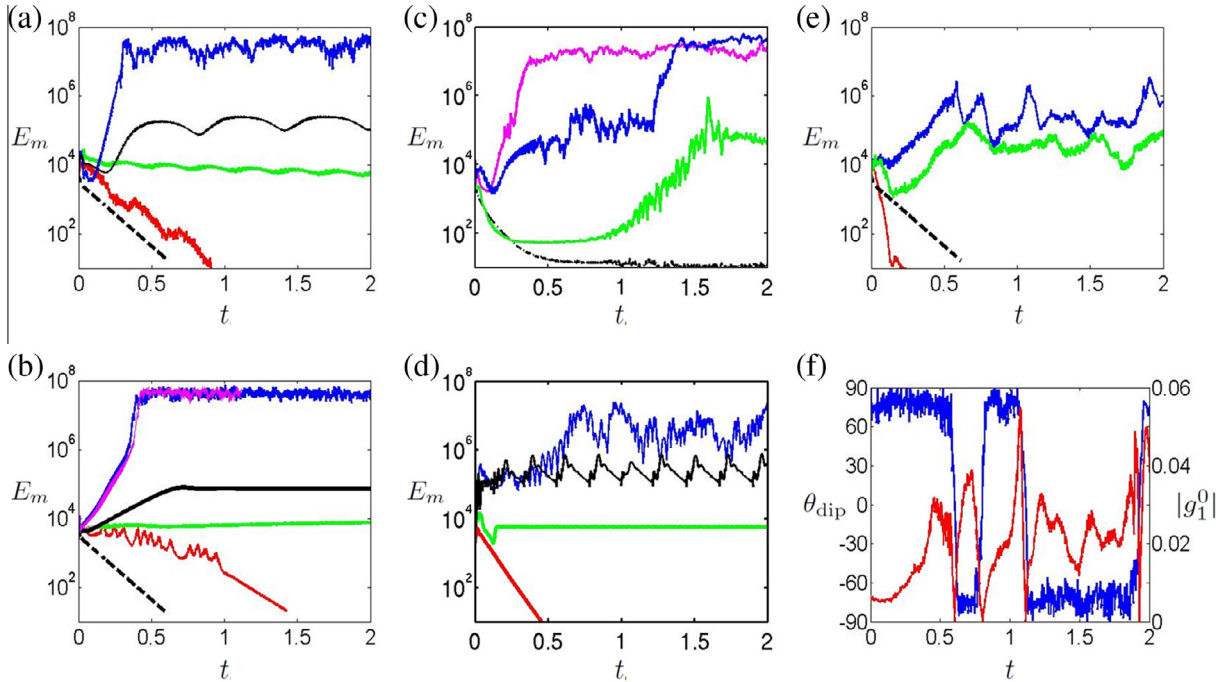


Fig. 4. (a)–(e) Magnetic energy E_m versus time t in units of magnetic diffusion time in the dynamo models with different patterns of lateral heterogeneity imposed on the CMB. The dashed black line in each graph is the reference model with homogeneous CMB heat flux that does not generate a dynamo. (a), (b) and (e) are runs with the parameters $E = 1.2 \times 10^{-4}$, $Pm = 50$ and $Ra/Ra_c = 1.2$, (c) is run with $E = 1.2 \times 10^{-5}$, $Pm = 10$ and $Ra/Ra_c = 1.8$ and (d) is run with $E = 1.2 \times 10^{-4}$, $Pm = 50$ and $Ra/Ra_c \sim 0.94$. (a) Y_1 with $q^* = 20\%$ (red), 30% (green), 35% (black) and 40% (blue). (b) Y_2 with $q^* = 10\%$ (red), 30% (green), 36% (black), 50% (blue) and 50% & stress-free inner boundary (magenta). (c) Y_1 with $q^* = 20\%$ (green), 40% (blue) and 60% (magenta). (d) Y_1 with $q^* = 20\%$ (red), 40% (green), 55% (black) and 60% (blue). (e) Y_1 with $q^* = 40\%$ (red), 55% (green) and 60% (blue). (f) Dipole tilt (left-hand side axis, blue line) and absolute value of the axial dipole Gauss coefficient (right-hand side axis, red line) for $q^* = 60\%$ in (e). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1

Summary of numerical dynamos with various imposed lateral CMB heat flux variations, for $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 1.2$. The dimensionless magnetic field Λ (Elsasser number) is obtained as a volume-averaged value from the model, E_m is the magnetic energy, E_k is the kinetic energy, Rm is the magnetic Reynolds number obtained from the root mean square velocity, ΔT_H is the horizontal variation in temperature at the outer boundary, ΔT_V is the vertical temperature difference and H_R (defined by Eq. (7)) is the relative axial (z) helicity calculated for one hemisphere. The letters NM and M refer to the non-magnetic and magnetic (saturated dynamo) states. Runs that produce $E_m/E_k \sim 10$ and $\Lambda \gtrsim 1$ are considered successful dynamos, whereas runs that produce $E_m/E_k \lesssim 1$ and $\Lambda \lesssim 1$ are considered 'marginal'. Note that all equatorially anti-symmetric cases produce failed or marginal dynamos, except in cases where polarity reversals occur.

Y_l^m (q°)	Λ	E_m/E_k	Rm	$\Delta T_H/\Delta T_V$	H_R (NM)	H_R (M)	Dynamo?	Reversal?
Y_0^0	–	–	379	0.07	–0.0341	–	No	–
Y_1^1 (20%)	–	–	411	0.16	–0.3277	–	No	–
Y_1^1 (30%)	–	–	494	0.25	–0.4548	–	No	–
Y_1^1 (35%)	0.39	0.225	528	0.31	–0.4145	–0.3665	Marginal	No
Y_1^1 (40%)	5.78	40.0	590	0.43	+0.0271	+0.6784	Yes	No
Y_1^1 (60%)	5.41	23.33	723	0.57	+0.6992	+0.7161	Yes	No
Y_2^2 (10%)	–	–	391	0.10	–0.1204	–	No	–
Y_2^2 (30%)	–	–	465	0.22	–0.1654	–	No	–
Y_2^2 (36%)	0.26	0.114	494	0.24	–0.1032	–0.1055	Marginal	No
Y_2^2 (50%)	6.13	50.0	560	0.33	–0.0811	+0.6322	Yes	No
Y_2^2 (60%)	5.93	42.0	590	0.43	+0.0005	+0.8346	Yes	No
Y_2^2 (90%)	5.64	34.5	619	0.59	+0.0105	+0.4249	Yes	No
Y_2^1 (40%)	–	–	446	0.40	–0.1298	–	No	–
Y_2^1 (55%)	0.22	0.09	483	0.50	–0.1191	–0.1040	Marginal	No
Y_2^1 (60%)	0.14–1.67	0.03–12.7	511	0.51	–0.1258	–0.0974	Yes	Yes
Y_2^1 (65%)	0.12–1.85	0.02–13.7	528	0.55	–0.1546	–0.1577	Yes	Yes
Y_2^1 (72%)	0.41	0.235	544	0.60	–0.1453	–0.1843	Marginal	No
Y_2^1 (80%)	–	–	567	0.64	–0.1548	–	No	–
Y_3^2 (45%)	–	–	419	0.29	–0.1364	–	No	–
Y_3^2 (60%)	0.57	1.091	438	0.37	–0.0612	–0.0925	Marginal	No
Y_3^2 (80%)	0.35	0.253	447	0.46	–0.1231	–0.1087	Marginal	No
Y_3^2 (85%)	–	–	512	0.50	–0.0477	–	No	–

Table 2

Summary of numerical dynamos with imposed Y_1^1 CMB heat flux variations, for $E = 1.2 \times 10^{-5}$, $Pr = 1$, $Pm = 10$ and $Ra/Ra_c = 1.8$. The column headings are the same as in Table 1.

Y_l^m (q°)	Λ	E_m/E_k	Rm	$\Delta T_H/\Delta T_V$	H_R (NM)	H_R (M)	Dynamo?	Reversal?
Y_0^0	0.002	6×10^{-5}	297	0.09	–0.0059	–	No	–
Y_1^1 (20%)	0.21	0.238	365	0.16	–0.0212	–0.0075	Marginal	No
Y_1^1 (40%)	3.42	64.152	390	0.37	+0.0916	+0.5375	Yes	No
Y_1^1 (60%)	2.94	46.114	403	0.47	+0.3724	+0.2587	Yes	No

Table 3

Summary of numerical dynamos with imposed Y_1^1 CMB heat flux variations, for $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 0.94$. The column headings are the same as in Table 1.

Y_l^m (q°)	Λ	E_m/E_k	Rm	$\Delta T_H/\Delta T_V$	H_R (NM)	H_R (M)	Dynamo?	Reversal?
Y_0^0	–	–	0	0.05	–	–	–	–
Y_1^1 (20%)	–	–	230	0.14	–0.1768	–	No	–
Y_1^1 (40%)	0.07	0.011	432	0.37	–0.0514	–0.2467	No	–
Y_1^1 (55%)	0.47	0.258	593	0.46	+0.4841	+0.5920	Marginal	No
Y_1^1 (60%)	2.39	4.148	757	0.55	+0.5736	+0.9871	Yes	No

must be practically “non-magnetic”. To mimic negligible back reaction of the magnetic field on the flow, for every dynamo run we performed an equivalent non-magnetic rotating convection run. The non-magnetic run with homogeneous outer boundary heat flux provides the reference against which comparisons can be made. Fig. 6(a) and (b) show the flow in the reference model, dominated by columnar vortices parallel to the axis of rotation (Busse, 1970). For the Y_1^1 heat flux variation at 60%, the axial flow (Fig. 6(c)) is as expected confined to the Western hemisphere where the boundary condition enhances the mean outward heat flux; furthermore, a strong contrast develops between flows going away and towards the equator. The radial flow (Fig. 6(d)) is domi-

nated by an isolated downwelling patch whose intensity is much higher than that in the reference model. The Y_2^2 variation, on the other hand, breaks the equatorial symmetry of the axial flow in rotation. The flows in Fig. 6(e) and (f), very similar to those obtained during reversals of the axial dipole in the dynamo models with the same heat flux pattern, are weaker than those in the reference model as they take skewed paths that extend all the way from the boundary of one hemisphere to the other, but in opposite directions. The fact that north-bound and south-bound flows do not differ much in strength has important consequences for the magnetic field produced by these flows: In the anti-symmetric CMB variation models, the axial dipole field intensity is much

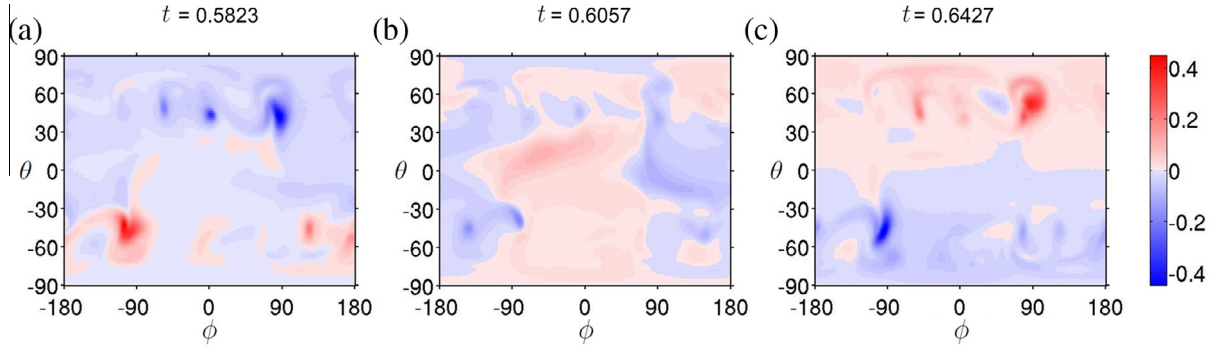


Fig. 5. Radial magnetic field at the CMB at three snapshots during a polarity reversal for the Y_2^1 heat flux pattern ($q^* = 60\%$). The dynamo parameters are $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 1.2$. (For interpretation to color in this figure, the reader is referred to the web version of this article.)

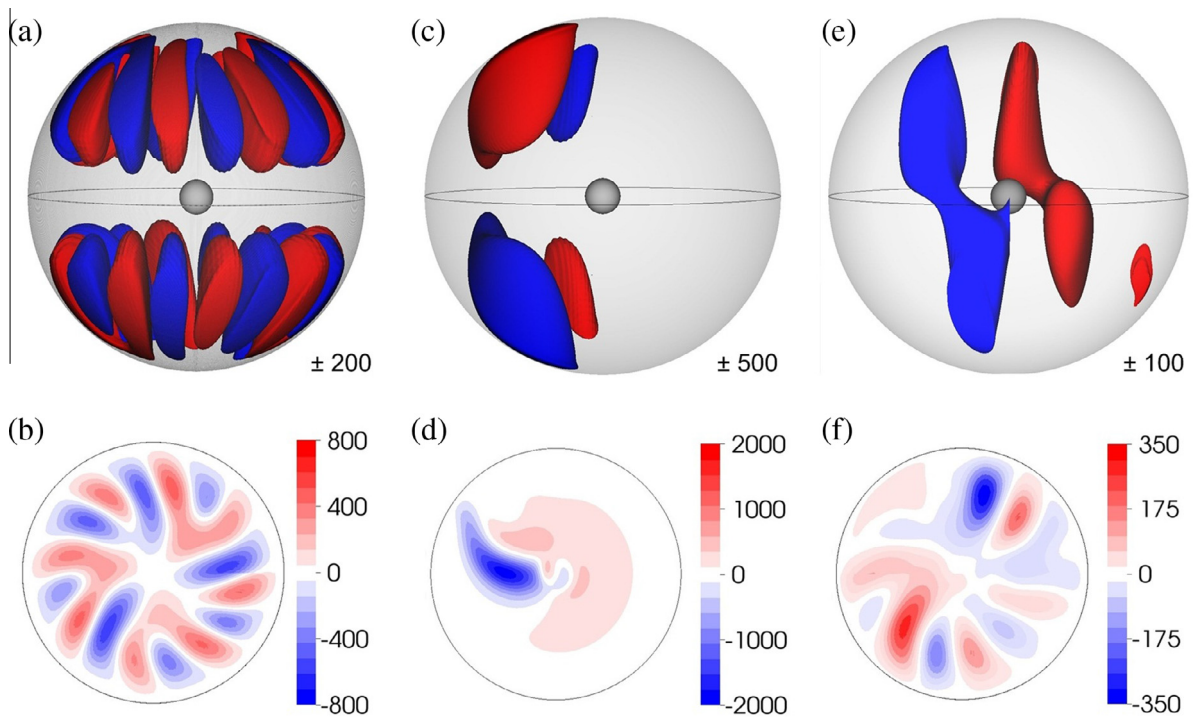


Fig. 6. Flows in the non-magnetic calculations at $E = 1.2 \times 10^{-4}$, $Pr = 1$ and $Ra/Ra_c = 1.2$. (a), (c) and (e) are isosurfaces of the axial (z) velocity with respective contour levels. (b), (d) and (f) are contour plots of the radial (r) velocity at the section $z = 0.2$ above the equator. (a) and (b): Homogeneous CMB heat flux; (c) and (d): Y_1^1 ($q^* = 60\%$); (e) and (f): Y_2^1 ($q^* = 60\%$). (For interpretation to color in this figure, the reader is referred to the web version of this article.)

weaker (compared to symmetric variation models), with significant secular variation in the form of mobility of the flux patches as well as polarity reversals. Furthermore, a larger q^* for anti-symmetric patterns ($\geq 80\%$ for Y_2^1 ; $\geq 85\%$ for Y_3^2) causes the dynamo to fail (Table 1), which limits their range of operation.

For $E = 1.2 \times 10^{-4}$ and $Ra/Ra_c = 0.94$, the reference model has no convection (Fig. 2(b)), but a Y_1^1 heat flux pattern with $q^* = 60\%$ triggers convection in the hemisphere of enhanced boundary heat flux, producing a flow that is very similar to that at $Ra/Ra_c = 1.2$ (Fig. 6(c) and (d)). The fact that this boundary-driven flow supports a strong magnetic field (Table 3) implies that equatorially symmetric CMB variations can support dynamo action in Earth even when the base state power sources are absent. The regime of $Ra/Ra_c \ll 1$ is not examined in this paper, and will be the subject of a separate study.

For convection with homogeneous outer boundary thermal conditions, rapid rotation produces cyclonic and anticyclonic vortices

(of positive and negative axial vorticity) aligned with the axis of rotation, with no difference between rising and sinking fluid motion (Busse, 1970; Olson et al., 1999). However, outer boundary variations in the form of low and high heat flux structures equispaced in longitude, as in our Y_1^1 variation, produce different flow intensities in cyclonic and anticyclonic vortices. This is visible in the difference between the axial kinetic helicity contained in anticyclones (H_A) and cyclones (H_C) in the non-magnetic simulations, measured by the relative helicity H_R (Sreenivasan et al., 2014):

$$H_R = \frac{H_A - H_C}{H_A + H_C}. \quad (7)$$

The values of H_R for the non-magnetic (NM) runs at 1.2×10^{-4} and $Ra/Ra_c = 1.2$ are given in Table 1. For the Y_1^1 case with small q^* , H_C exceeds H_A so that H_R is negative and the field is weak (or no dynamo is excited). With $q^* = 40\%$, H_A marginally exceeds H_C so that $H_R = +0.027$; when this holds, the seed field in the correspond-

ing dynamo run grows. Further increase of q^* consistently produces positive H_R and strong-field dynamos. The case with 50% Y_2^1 variation is equivalent ($H_A \approx H_C$), and a strong magnetic field is generated. In summary, if the heterogeneous outer boundary heat flux in the non-magnetic run produces positive relative helicity, dynamo action is expected in the equivalent magnetic run. In the cases with equatorially anti-symmetric lateral variations, we find that $H_A < H_C$; consequently these CMB heat flux patterns consistently produce magnetic fields that have small axial dipole intensity and in some cases reverse. These results imply that the equatorial symmetry of the CMB heat flux may determine the strength and stability of the magnetic field under weak thermal convection.

Fig. 7 clarifies the picture developed so far for dynamo action in our models. We have focused on the initial growth phase of the magnetic field rather than the saturated phase of the dynamo. If the boundary heat flux is homogeneous, there is no difference in strength between cyclonic and anticyclonic vortices (Fig. 7(a) and (b)). For the 60% Y_1^1 variation, a pair of oppositely signed vortices clearly develop, with more intense axial flow in the anticyclones. Note that the difference in vorticity magnitude itself is not much between the columns when dynamo action sets in; it is the axial flow intensity and the width of the vortex that are different (Fig. 7(c) and (d) and Table 4). The marginal dominance of anticyclonic helicity for the 40% Y_1^1 variation (Table 1) is not visible in the

Table 4

Axial kinetic energy distribution in the Northern hemisphere (all values per cent) from non-magnetic simulations at $E = 1.2 \times 10^{-4}$, $Pr = 1$ and $Ra/Ra_c = 1.2$. Here $E_z = \frac{1}{2} \int u_z^2 dV$ is the total axial kinetic energy density, $E_{z+} = \frac{1}{2} \int u_{z+}^2 dV$ is the axial kinetic energy made up of only positive axial velocity and E_{z-} is the axial kinetic energy made up of only negative axial velocity. E_{zA} and E_{zC} are the kinetic energies contained in anticyclonic and cyclonic vortices.

$Y_l^m (q^*)$	E_{z+}/E_z	E_{z-}/E_z	E_{zA}/E_z	E_{zC}/E_z
Y_0^0	52.2	47.8	50.9	49.1
Y_1^1 (60%)	78.8	21.2	67.4	32.6
Y_2^1 (55%)	48.4	51.6	48.3	51.7

volume plot (Fig. 8(a) and (b)), but the difference for the 60% case becomes clear (Fig. 8(c) and (d)). The anti-symmetric Y_2^1 case, on the other hand, does not exhibit any clear difference between H_A and H_C (Fig. 8(e) and (f)), consistent with the fact that this pattern produces weak and reversing magnetic fields.

4. The impact of outer boundary heterogeneity on core flow and the magnetic field

If the back reaction of the small seed magnetic field on the flow is negligible, the main (curled) force balance in the shell on time and volume average is between the buoyancy and Coriolis forces, which is termed the thermal wind equation (e.g. Pedlosky, 1987):

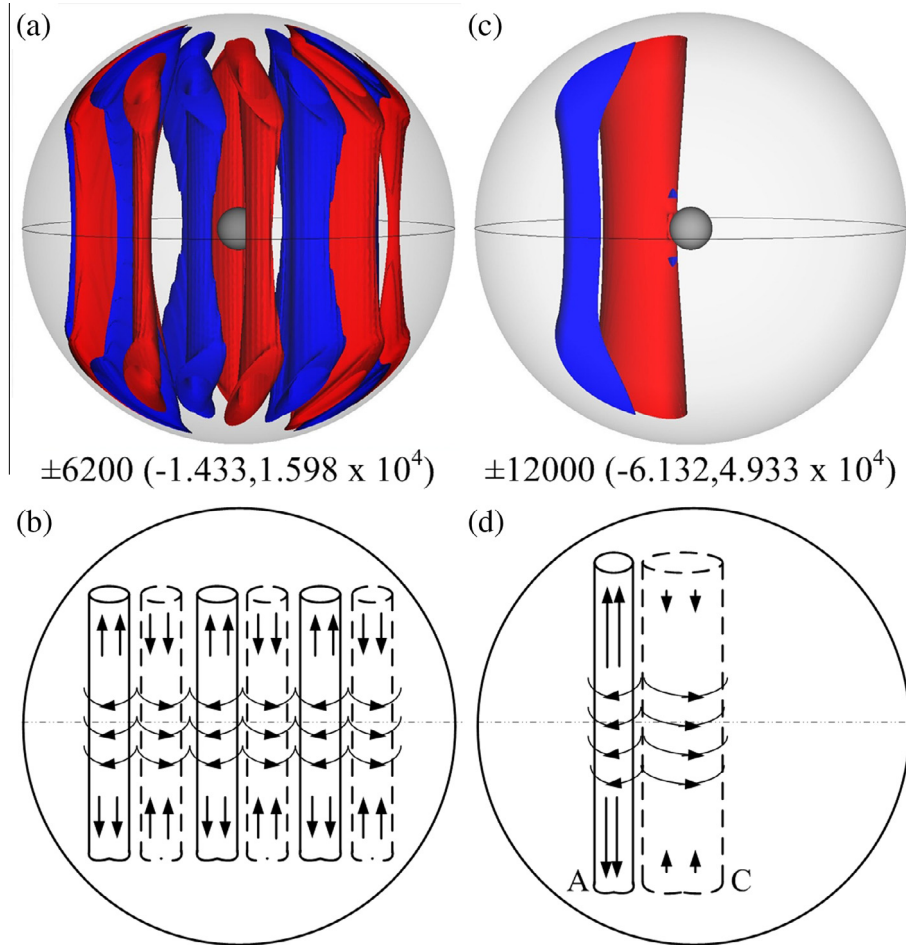


Fig. 7. (a) Isosurfaces of the axial vorticity ω_z for the reference model with homogeneous CMB heat flux. Positive (cyclonic) vorticity is shown in red and negative (anticyclonic) vorticity is in blue. (b) A cartoon of this model illustrating positive and negative columnar vortices with axial flow arrows superposed. (c) Isosurfaces of ω_z for the 60% Y_1^1 variation. (d) Cartoon showing a cyclone-anticyclone pair for the run of (c), with the axial flow intensity (represented by the lengths of the arrows) in the anticyclone (A) greater than that in the cyclone (C). The contour values are given in the isosurface plots (with peak values in the volume in brackets). The runs are non-magnetic, with parameters $E = 1.2 \times 10^{-4}$, $Pr = 1$ and $Ra/Ra_c = 1.2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

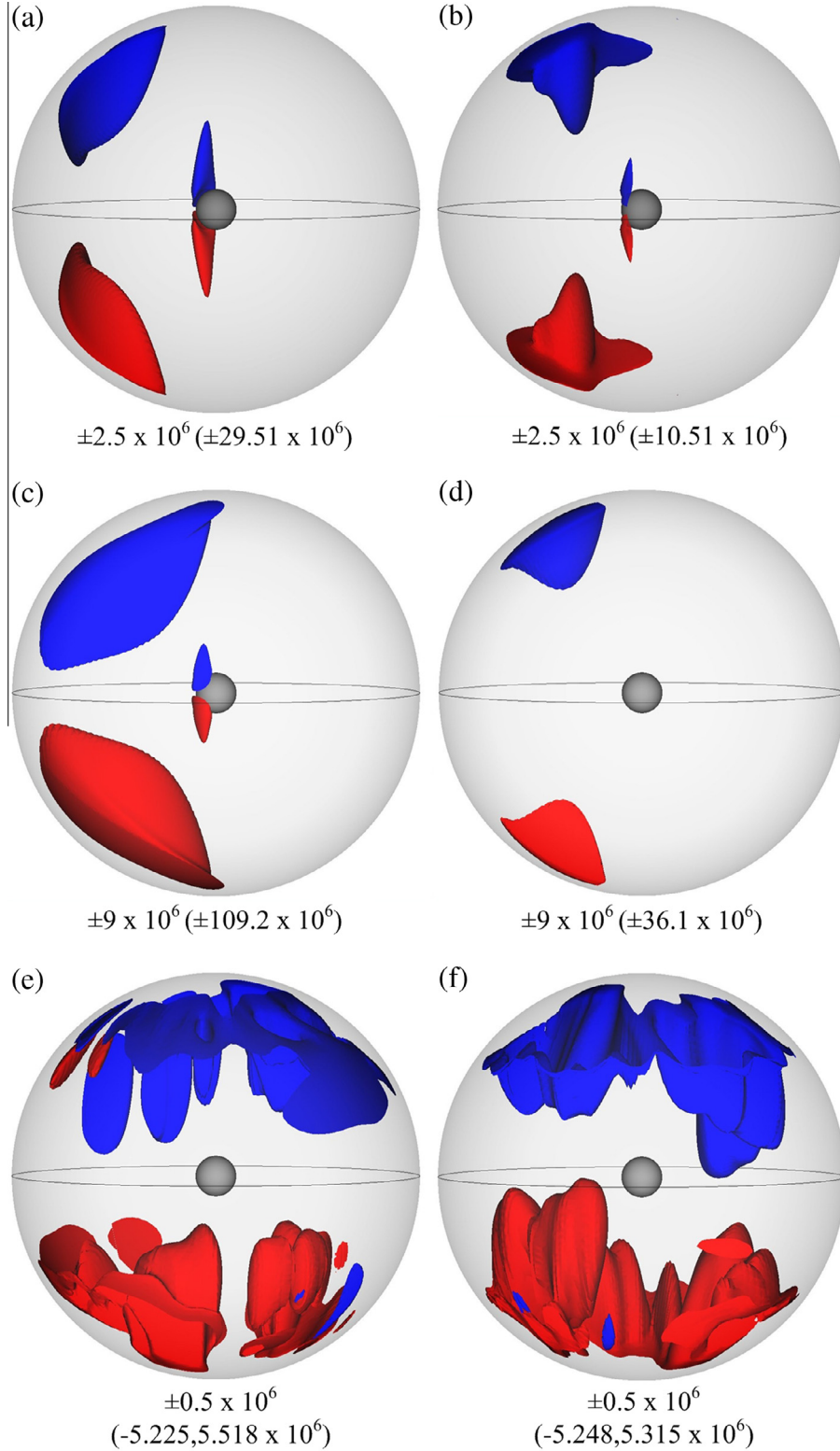


Fig. 8. (a) and (b): Time-averaged isosurfaces of the axial part of the kinetic helicity, H for the Y_1^1 run at $q^* = 40\%$, with anticyclonic helicity H_A in (a) and cyclonic helicity H_C in (b). (c) and (d): H_A and H_C for Y_1^1 ($q^* = 60\%$). (e) and (f): H_A and H_C for Y_2^2 ($q^* = 60\%$). All plots are obtained from non-magnetic runs that use $E = 1.2 \times 10^{-4}$, $Pr = 1$ and $Ra/Ra_c = 1.2$. The contour values are given in the plots (with peak values in the volume in brackets). (For interpretation to color in this figure, the reader is referred to the web version of this article.)

$$\frac{\partial \mathbf{u}}{\partial z} + PmPr^{-1} Ra \nabla \times (\mathbf{T}\mathbf{r}) = 0. \quad (8)$$

As rapidly rotating convection takes the form of columnar structures aligned with the rotation axis, we are particularly interested in the axial (z) component of (8), given by

$$\frac{\partial u_z}{\partial z} - PmPr^{-1} Ra \frac{1}{s} \frac{\partial T}{\partial \phi} = 0 \quad (9)$$

in cylindrical polar coordinates (s, ϕ, z) . For a positive (negative) temperature gradient in ϕ , the Coriolis effect enhances flow in anti-

cyclones (cyclones), causing anticyclonic (cyclonic) kinetic helicity H_A (H_C) to dominate. From the axial kinetic energy ratios given in Table 4, we find that the reference model with homogeneous outer boundary heat flux (Y_0^0) maintains approximate parity between positive (axially upward) and negative (axially downward) fluid velocity, as well as between the energy contained in anticyclones and cyclones. In contrast, the equatorially symmetric Y_1^1 pattern produces significantly higher positive velocity in the Northern hemisphere and higher energy in the anticyclones. The anti-symmetric Y_2^1 pattern, however, does not produce any marked difference between cyclonic and anticyclonic flows for $q^* < 80\%$, as in the homogeneous reference case.

Fig. 9 provides further insight into the dynamics with equatorially symmetric and anti-symmetric heat flux patterns. The Y_1^1 pattern interacts with background convection in the thermal wind equation in a way that creates an augmented positive $\partial u_z / \partial z$ (Fig. 9(b)), which renders the axial flow (and helicity) in anticyclones larger in magnitude than in cyclones. On the other hand, the anti-symmetric Y_2^1 pattern is unable to interact with background convection in this fashion (Fig. 9(c) and (d)); consequently, H_A never exceeds H_C .

The equatorially symmetric boundary conditions that favour anticyclonic helicity also support strong axial dipole field intensities (Fig. 10(a) and (b)), whereas the anti-symmetric boundary conditions consistently produce axial dipole field intensities at least one order of magnitude smaller (Fig. 10(c) and (f)). Furthermore, the Y_2^1 case at $q^* = 65\%$ shows reversals of the axial dipole, as in $q^* = 60\%$ (Fig. 4(f)). We therefore conclude that the helicity distribution with anti-symmetric boundary conditions does not sustain magnetic fields with dominant axial dipoles.

5. Discussion

In this study, we considered synthetic, large-scale heat flux patterns of both symmetries about the equator that likely contributed to any CMB heterogeneity throughout Earth's history. Both equatorially symmetric and anti-symmetric heat flux patterns generate a dynamo, but the latter is confined to a more limited range of heterogeneity amplitudes. In addition, symmetric patterns tend to promote non-reversing dynamos (Pétrellis et al., 2009, 2011), whereas anti-symmetric patterns produce dynamos with weak axial dipole fields and in some cases, polarity reversals.

The magnetic Reynolds number in the dynamo, which has been traditionally used as a measure of dynamo action, generally increases with q^* , but does not help determine a threshold for dynamo onset. In addition, reversals that typically occur in dynamo models with strong convection (Christensen and Aubert, 2006; Olson and Christensen, 2006; Aubert et al., 2009) are present in our weakly convecting models. Both these findings may be explained based on the difference in kinetic helicity between cyclonic and anti-cyclonic columnar vortices. When anticyclonic helicity dominates over cyclonic helicity a dynamo exists, whereas when the two helicities are comparable the field is weak and the dynamo is bound to fail even at large Rm . An equally remarkable result is that an equatorially antisymmetric boundary heat flux pattern produces polarity reversals in a marginally supercritical ($Ra \sim Ra_c$) regime because anticyclonic helicity never exceeds cyclonic helicity. Favourable conditions for positive relative helicity H_R and non-reversing dynamo onset appear only for symmetric heat flux patterns with increasing q^* . Obtaining $H_R > 0$ in a strongly time-varying, $Ra \gg Ra_c$ regime does not require any support from boundary heat flux variations; rather, it is in the

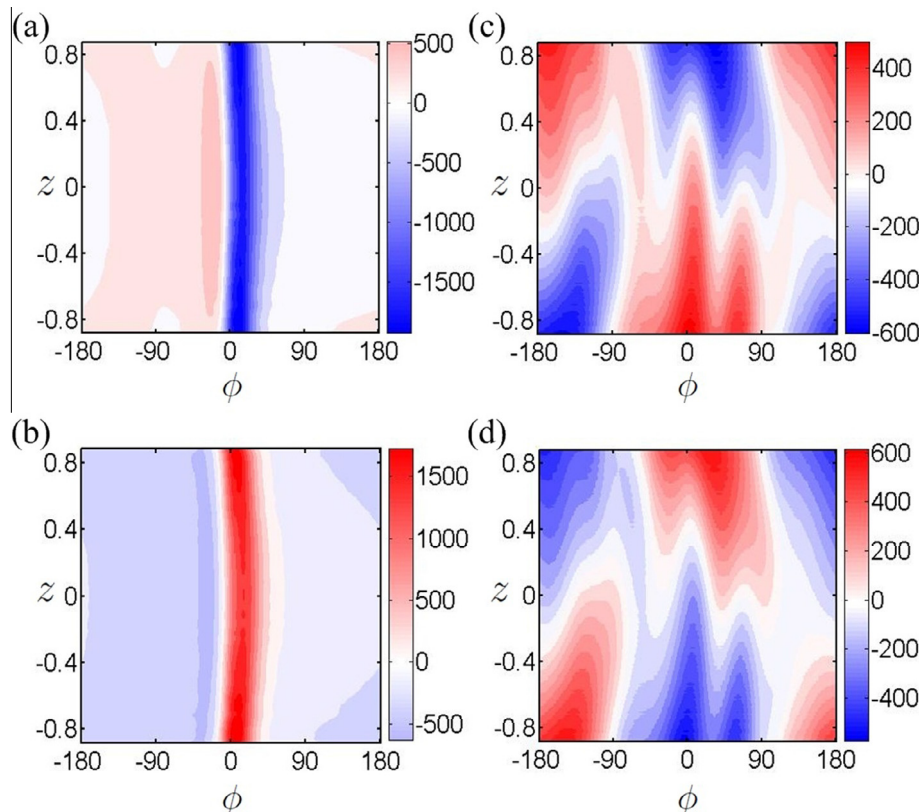


Fig. 9. Buoyancy (top panels) and Coriolis (bottom panels) terms in Eq. (9) shown on a cylindrical ($z - \phi$) section of radius $s = 0.45$. (a) and (b) Y_1^1 ($q^* = 60\%$); (c) and (d) Y_2^1 ($q^* = 55\%$). The runs are non-magnetic, with parameters $E = 1.2 \times 10^{-4}$, $Pr = 1$ and $Ra/Ra_c = 1.2$. (For interpretation to color in this figure, the reader is referred to the web version of this article.)

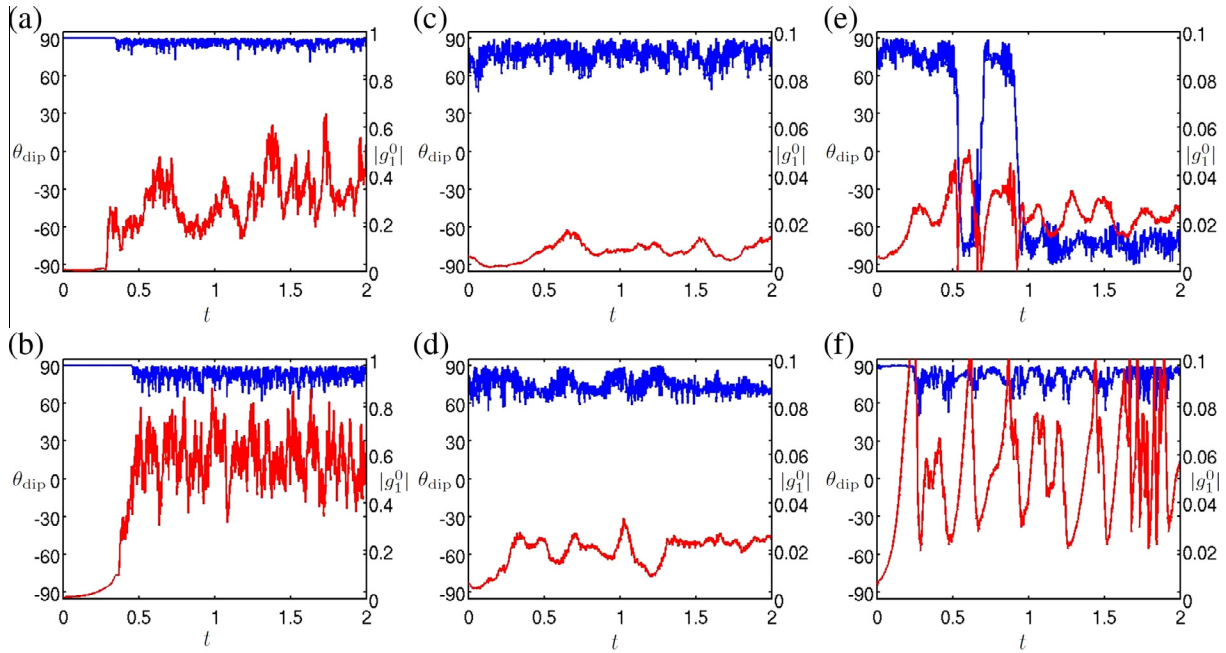


Fig. 10. Dipole tilt θ_{dip} in degrees (left-hand side axis, blue line) and the absolute value of the axial dipole Gauss coefficient $|g_1^0|$ (right-hand side axis, red line) versus time t in units of magnetic diffusion time for equatorially symmetric and anti-symmetric CMB heterogeneities. (a) Y_1^1 (40%), (b) Y_2^2 (50%), (c) Y_2^1 (55%), (d) Y_2^1 (72%), (e) Y_2^1 (65%) and (f) Y_2^2 (80%). Note the scale differences for $|g_1^0|$ between the (a)–(b) and (c)–(f) cases. The dynamo parameters are $E = 1.2 \times 10^{-4}$, $Pr = 1$, $Pm = 50$ and $Ra/Ra_c = 1.2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$Ra \sim Ra_c$ regime that these variations play a crucial role in establishing $H_R > 0$. We recall that the core–mantle coupling effect is much stronger for $Ra \sim Ra_c$ than for $Ra \gg Ra_c$ (Sreenivasan and Gubbins, 2008).

We note that there is an additional enhancement in the relative helicity that develops in rapidly rotating, dipolar dynamos because of the back reaction of the magnetic field on the flow (Sreenivasan et al., 2014). From Tables 1–3 it is evident that the relative helicity for the saturated dynamo is in general greater than that for the corresponding non-magnetic run (with equatorially symmetric CMB heat flux patterns). The field-generated relative helicity is indeed an important effect in dipole-dominated dynamos and bears analogy to the equatorially symmetric boundary heterogeneity control. It is, however, secondary for the growth of a small seed magnetic field in the Earth's core.

Because boundary heterogeneity plays a major role in the dynamics of our models, it is important to verify that the Boussinesq approximation, which the models rely on, is not violated in the dynamo models with heterogeneous CMB heat flux. In Tables 1–3 we report the ratios of lateral to vertical temperature differences ($\Delta T_H/\Delta T_V$) in the models. In all cases the ratios are smaller than unity, implying that density variations introduced by the boundary heterogeneity are not likely to exceed that produced by thermal buoyancy.

Any geophysical application of our results, mainly for early Earth conditions, should be taken with caution, due to the unrealistic control parameters of our dynamo models. In particular, our Ekman number E and magnetic Prandtl number Pm are much larger than their values in the core. The large Pm (10–50) makes dynamo action possible at $Ra \sim Ra_c$ and $q^* < 100\%$. That said, our simulations at $E = 1.2 \times 10^{-5}$ suggest that the effect of CMB heat flux heterogeneity on dynamo onset may persist at lower E and Pm values.

The Rayleigh number for thermal convection in early Earth's core is not well constrained. Estimates of the thermal Rayleigh number based on the turbulent diffusivity suggest that convection in the core is not highly supercritical (Gubbins, 2001; Anufriev

et al., 2005), possibly within the regime where current geodynamo models operate. Here we find that symmetric boundary heat flux patterns produce dynamo onset even below the threshold for thermal convection ($Ra < Ra_c$) in the reference model. This result is consistent with linear onset studies where boundary-driven thermal winds yield convection at lower Rayleigh numbers (Teed et al., 2010). Whether the symmetric heat flux patterns can drive flows that excite the dynamo for $Ra \ll Ra_c$ is not known. However, it seems sensible that increasing q^* together with decreasing Ra would give dynamos.

Our findings may resolve the enigmatic problem of how the geodynamo was maintained under a tight energy budget prior to inner core nucleation. The influence of lower mantle heterogeneity on the geodynamo may be much more fundamental than merely affecting the morphologies of the geomagnetic field and core flow (Aubert et al., 2008; Gubbins et al., 2011): These CMB thermal anomalies may power the geodynamo in the absence of compositional convection. On the other hand, if $Ra > Ra_c$ in early Earth as recent studies suggest (Driscoll and Bercovici, 2014; Nakagawa and Tackley, 2014; Davies, 2015; Labrosse, 2015), the CMB heat flux pattern would strengthen the magnetic field generated by thermal convection in the core.

Finally, the back and forth transitions from hyper-reversing periods to superchrons observed in paleomagnetic records (Biggin et al., 2012; Gradstein et al., 2012) may represent changes in the degree of equatorial symmetry in the lateral thermal distribution of the lowermost mantle. In addition, our results suggest that in order to sustain a strong geomagnetic field, the lower mantle thermal distribution was likely dominantly symmetric about the equator.

Acknowledgment

Binod Sreenivasan thanks the Department of Science and Technology (Government of India) for the award of a SwarnaJayanti Fellowship.

References

- Amit, H., Choblet, G., 2009. Mantle-driven geodynamo features – effects of post-perovskite phase transition. *Earth Planets Space* 61, 1255–1268.
- Amit, H., Aubert, J., Hulot, G., 2010. Stationary, oscillating or drifting mantle-driven geomagnetic flux patches? *J. Geophys. Res. B* 07108. <http://dx.doi.org/10.1029/2009JB006542>.
- Anufriev, A.P., Jones, C.A., Soward, A.M., 2005. The Boussinesq and anelastic liquid approximations for convection in the earth's core. *Phys. Earth Planet. Inter.* 152, 163–190.
- Aubert, J., Amit, H., Hulot, G., 2007. Detecting thermal boundary control in surface flows from numerical dynamos. *Phys. Earth Planet. Inter.* 160, 143–156.
- Aubert, J., Amit, H., Hulot, G., Olson, P., 2008. Thermochemical flows couple the Earth's inner core growth to mantle heterogeneity. *Nature* 454, 758–761.
- Aubert, J., Labrosse, S., Poitou, C., 2009. Modelling the paleo-evolution of the geodynamo. *Geophys. J. Int.* 179, 1414–1428.
- Aurnou, J., Aubert, J., 2011. End-member models of boundary-modulated convective dynamos. *Phys. Earth Planet. Inter.* 187, 355–363.
- Biggin, A.J., Steinberger, B., Aubert, J., Suttie, N., Holme, R., Torsvik, T.H., van der Meer, D.G., van Hinsbergen, D.J.J., 2012. Possible links between long-term geomagnetic variations and whole-mantle convection processes. *Nat. Geosci.* 5, 526–533.
- Bloxham, J., 2002. Time-independent and time-dependent behaviour of high-latitude flux bundles at the core-mantle boundary. *Geophys. Res. Lett.* 29. <http://dx.doi.org/10.1029/2001gl014543>.
- Busse, F.H., 1970. Thermal instabilities in rapidly rotating systems. *J. Fluid Mech.* 44, 441–460.
- Christensen, U., Aubert, J., 2006. Scaling properties of convection-driven dynamos in rotating spherical shells and application to planetary magnetic fields. *Geophys. J. Int.* 166, 97–114.
- Davies, C.J., 2015. Cooling history of earth's core with high thermal conductivity. *Phys. Earth Planet. Inter.* <http://dx.doi.org/10.1016/j.pepi.2015.03.007>.
- de Koker, N., Steinle-Neumann, G., Vojtech, V., 2012. Electrical resistivity and thermal conductivity of liquid Fe alloys at high P and T and heat flux in Earth's core. *Proc. Natl. Acad. Sci.* 109, 1070–1073.
- Dietrich, W., Wicht, J., 2013. A hemispherical dynamo model: implications for the Martian crustal magnetization. *Phys. Earth Planet. Inter.* 217, 10–21.
- Driscoll, P., Bercovici, D., 2014. On the thermal and magnetic histories of earth and venus: influences of melting, radioactivity, and conductivity. *Phys. Earth Planet. Inter.* 236, 36–51.
- Glatzmaier, G., Coe, R., Hongre, L., Roberts, P., 1999. The role of the earth's mantle in controlling the frequency of geomagnetic reversals. *Nature* 401, 885–890.
- Gradstein, F., Ogg, J., Schmitz, M., Ogg, G., 2012. *The Geologic Time Scale 2012*. Elsevier Science, Amsterdam.
- Gubbins, D., 2001. The Rayleigh number for convection in the Earth's core. *Phys. Earth Planet. Inter.* 128, 3–12.
- Gubbins, D., Willis, A.P., Sreenivasan, B., 2007. Correlation of Earth's magnetic field with lower mantle thermal and seismic structure. *Phys. Earth Planet. Inter.* 162, 256–260.
- Gubbins, D., Sreenivasan, B., Mound, J., Rost, S., 2011. Melting of the Earth's inner core. *Nature* 473, 361–363.
- Hirose, K., Labrosse, S., Hernlund, J., 2013. Compositional state of Earth's core. *Ann. Rev. Earth Planet. Sci.* 41, 657–691.
- Hori, K., Wicht, J., Christensen, U.R., 2010. The effect of thermal boundary conditions on dynamos driven by internal heating. *Phys. Earth Planet. Inter.* 182, 85–97. <http://dx.doi.org/10.1016/j.pepi.2010.06.011>.
- Kono, M., Roberts, P.H., 2002. Recent geodynamo simulations and observations of the geomagnetic field. *Rev. Geophys.* 40, 4.
- Kutzner, C., Christensen, U.R., 2004. Simulated geomagnetic reversals and preferred virtual geomagnetic pole paths. *Geophys. J. Int.* 157, 1105–1118.
- Labrosse, S., 2015. Thermal evolution of the core with a high thermal conductivity. *Phys. Earth Planet. Inter.* <http://dx.doi.org/10.1016/j.pepi.2015.02.002>.
- Landeau, M., Aubert, J., 2011. Equatorially asymmetric convection inducing a hemispherical magnetic field in rotating spheres and implications for the past martian dynamo. *Phys. Earth Planet. Inter.* 185 (3), 61–73.
- Moffatt, H.K., 1978. *Magnetic Field Generation in Electrically Conducting Fluids*. Cambridge University Press, Cambridge, U.K.
- Nakagawa, T., Tackley, P.J., 2013. Implications of high core thermal conductivity on earth's coupled mantle and core evolution. *Geophys. Res. Lett.* 40 (11), 2652–2656.
- Nakagawa, T., Tackley, P.J., 2014. Influence of combined primordial layering and recycled MORB on the coupled thermal evolution of Earth's mantle and core. *Geochim. Geophys. Geosyst.* 15 (3), 619–633.
- Olson, P., 2007. Overview. In: Olson, P. (Ed.), *Treatise on Geophysics*, vol. 8. Elsevier Science.
- Olson, P., Amit, H., 2014. Magnetic reversal frequency scaling in dynamos with thermochemical convection. *Phys. Earth Planet. Inter.* 229, 122–133.
- Olson, P., Christensen, U., 2002. The time averaged magnetic field in numerical dynamos with nonuniform boundary heat flow. *Geophys. J. Int.* 151, 809–823.
- Olson, P., Christensen, U., 2006. Dipole moment scaling for convection-driven planetary dynamos. *Earth Planet. Sci. Lett.* 250, 561–571.
- Olson, P., Christensen, U.R., Glatzmaier, G.A., 1999. Numerical modeling of the geodynamo: mechanisms of field generation and equilibration. *J. Geophys. Res.* 104, 10383–110404.
- Olson, P., Coe, R.S., Driscoll, P.E., Glatzmaier, G.A., Roberts, P.H., 2010. Geodynamo reversal frequency and heterogeneous core-mantle boundary heat flow. *Phys. Earth Planet. Inter.* 180, 66–79.
- Olson, P.L., Glatzmaier, G.A., Coe, R.S., 2011. Complex polarity reversals in a geodynamo model. *Earth Planet. Sci. Lett.* 304 (1), 168–179.
- Pedlosky, J., 1987. *Geophysical Fluid Dynamics*. Springer-Verlag, New York, USA.
- Pétreilis, F., Fauve, S., Dormy, E., Valet, J.P., 2009. Simple mechanism for reversals of Earth's magnetic field. *Phys. Rev. Lett.* 102, 144503.
- Pétreilis, F., Besse, J., Valet, J.P., 2011. Plate tectonics may control geomagnetic reversal frequency. *Geophys. Res. Lett.* 38, L19303.
- Pozzo, M., Davies, C., Gubbins, D., Alfè, D., 2012. Thermal and electrical conductivity of iron at Earth's core conditions. *Nature* 485, 355–358.
- Pozzo, M., Davies, C., Gubbins, D., Alfè, D., 2013. Transport properties for liquid silicon-oxygen-iron mixtures at Earth's core conditions. *Phys. Rev. B* 87, 014110.
- Sreenivasan, B., 2009. On dynamo action produced by boundary thermal coupling. *Phys. Earth Planet. Inter.* 177, 130–138.
- Sreenivasan, B., Gubbins, D., 2008. Dynamos with weakly convecting outer layers: implications for core-mantle boundary interaction. *Geophys. Astrophys. Fluid Dyn.* 102 (4), 395–407.
- Sreenivasan, B., Sahoo, S., Dhama, G., 2014. The role of buoyancy in polarity reversals of the geodynamo. *Geophys. J. Int.* 199 (3), 1698–1708.
- Tarduno, J.A., Cottrell, R.D., Davis, W.J., Nimmo, F., Bono, R.K., 2015. A Hadean to Paleoproterozoic geodynamo recorded by single zircon crystals. *Science* 349 (6247), 521–524.
- Teed, R.J., Jones, C.A., Hollerbach, R., 2010. Rapidly rotating plane layer convection with zonal flow. *Geophys. Astrophys. Fluid Dyn.* 104 (5–6), 457–480.
- Willis, A.P., Sreenivasan, B., Gubbins, D., 2007. Thermal core-mantle interaction: exploring regimes for 'locked' dynamo action. *Phys. Earth Planet. Inter.* 165, 83–92.